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Nonclassicality of a photon-subtracted Gaussian field

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We investigate the nonclassicality of a photon-subtracted Gaussian field, which was produced in a recent experiment, using negativity of the Wigner function and the nonexistence of well-behaved positive P function. We obtain the condition to see negativity of the Wigner function for the case including the mixed Gaussian incoming field, the threshold photodetection and the inefficient homodyne measurement. We show how similar the photon-subtracted state is to a superposition of coherent states.

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I. INTRODUCTION

The recent development of quantum optics has opened the possibility of generating and manipulating various nonclassical light fields, which cannot be described by classical theory, in a real laboratory. It is generally accepted that the presence of a positive well-defined P function (a quasiprobability function in phase space [3]) signals the field classical [4]; otherwise the field is categorized as nonclassical. A stronger constraint on nonclassicality is the presence of negativity in the Wigner function (another quasiprobability function) of the field [5]. While a Gaussian field may not have its P function, its Wigner function never becomes negative. For example, the squeezed vacuum state is represented by its Gaussian Wigner function while its P function does not exist [6]. It is also known that a Gaussian field remains Gaussian by linear transformations which correspond to basic tools in a quantum optics laboratory such as a phase shifter, a beam splitter, and a squeezer [1,2].

Two better-known nonclassical fields are a squeezed state and a superposition of two separate coherent states (coherent-state superposition). The two kinds of states are closely related to probably the most fundamental and intriguing paradoxes in quantum theory, i.e., the Einstein-Podolsky-Rosen paradox [7] for a two-mode squeezed state and the Schrödinger's cat paradox [8] for a coherent-state superposition. They are also known as useful resources for various schemes in quantum-information processing. A squeezed state and a coherent-state superposition manifest different types of nonclassicality. Whereas a squeezed state is a Gaussian field, a coherent-state superposition is non-Gaussian and shows a large amount of negativity in its Wigner function. There was an early attempt to relate the two states through quantum noise of arbitrary strength [9]. Dakna *et al.* [10] considered a connection between the two states by subtracting a precise number of photons from a squeezed field. They also showed that any quantum state can be generated from the vacuum by application of the coherent displacement operator and adding photons [11]. On the other hand, it has been reported that by squeezing a single-photon state one can generate a state which has almost unit fidelity to a coherent-state superposition of small amplitude [12].

It is only very recently that a traveling non-Gaussian field was experimentally generated by subtracting a photon from a squeezed vacuum by Wenger *et al.* [13]. They used a beam splitter and a threshold detector to subtract a photon from the squeezed field, but the reconstructed Wigner function failed to show a negative value [13]. It is thus timely to analyze the generation of a non-Gaussian state in relation to the status of experiments. In particular, as such the state forms a starting point for distillation of a continuous-variable field for quantum-information processing [14] and may improve the efficiency of quantum teleportation [15], the study will be of use. In this paper, we assess the nonclassicality of a photon-subtracted Gaussian field and study how similar this state is to a coherent-state superposition. We assess negativity of the Wigner function in conjunction with the nonexistence of the positive P function.

II. FIELD GENERATED BY SUBTRACTING A PHOTON

We would like to consider what kind of state one produces by eliminating one photon from a simple Gaussian function. A single-mode Gaussian field of its density operator $\hat{\rho}$ may be represented by the Weyl characteristic function [16–19] defined as $C(\xi)=\mathrm{Tr}[\hat{D}(\xi)\hat{\rho}]$:

$$C(\xi) = \exp\left(-\frac{A}{2}\xi_r^2 - \frac{B}{2}\xi_i^2\right), \tag{1}$$

where A and B are determined by the quadrature variances of the field. The displacement operator has been defined as $\hat{D}(\xi)=\exp(\xi\hat{a}^\dagger-\xi^*\hat{a})$, where $\hat{a}$ and $\hat{a}^\dagger$ are bosonic annihilation and creation operators, respectively. Note also that the density operator can be obtained from the characteristic function as

$$\hat{\rho} = \frac{1}{\pi}\int d^2\xi\, C(\xi)\hat{D}(-\xi), \tag{2}$$

which can be straightforwardly obtained using the identities $(1/\pi)\int d^2\alpha|\alpha\rangle\langle\alpha|=\mathbb{1}$ and [20]

$$|\alpha\rangle\langle\beta| = \frac{1}{\pi}\int d^2\xi\, \hat{D}(-\xi)\langle\beta|\hat{D}(\xi)|\alpha\rangle$$

where $|\alpha\rangle$ is a coherent state of amplitude α. Even though Eq. (1) does not represent a very general Gaussian field, rotation and/or displacement operation brings any Gaussian field to this form. It is useful to start with Eq. (1) because it is extremely challenging to produce a pure squeezed state with $AB=1$ and the characteristic function (1) allows us to treat a single-mode Gaussian state of a mixed state. The uncertainty relation is given by $AB\geqslant 1$ and the Gaussian state is called squeezed when either $A<1$ or $B<1$.

Let us consider the experiment by Wenger *et al.* [13]. First of all, they produce a squeezed Gaussian state; then this passes through a beam splitter with transmittivity $T=t^2$, where the other input port is assumed to be served by a vacuum. At the one output of mode 2, we conditionally measure a one-photon state $|1\rangle_2$. The state obtained at the other output port of mode 1 was what Wenger *et al.* produced as a non-Gaussian field in their experiment. We will evaluate the Wigner function for this field of mode 1.

By beam splitting the squeezed Gaussian field whose characteristic function is written as (1) and the vacuum of its characteristic function $C_v(\xi)=\exp\left(-\frac{1}{2}|\xi|^2\right)$, the characteristic function for the output field of modes 1 and 2 is [21]

$$C_{\mathrm{out}}(\eta,\xi)=\exp\left(-\frac{1}{2}\mathbf{x}\mathbf{V}\mathbf{x}^T\right) \tag{3}$$

where $\mathbf{x}=(\eta_r,\eta_i,\xi_r,\xi_i)$ and the correlation matrix

$$\mathbf{V}=\begin{pmatrix} n_1 & 0 & c_1 & 0\\ 0 & n_2 & 0 & c_2\\ c_1 & 0 & m_1 & 0\\ 0 & c_2 & 0 & m_2\end{pmatrix} \tag{4}$$

with

$$n_1=TA+R,\quad n_2=TB+R,\quad c_1=\mathrm{tr}(A-1),$$

$$c_2=\mathrm{tr}(B-1),\quad m_1=RA+T,\quad m_2=RB+T, \tag{5}$$

and $T=t^2$ and $R=r^2$.

We then use the two-mode version of Eq. (2) for the density operator of the output field:

$$\hat{\rho}_{\mathrm{out}}=\frac{1}{\pi^2}\int C_{\mathrm{out}}(\eta,\xi)\hat{D}_1(-\eta)\hat{D}_2(-\xi)d^2\eta\, d^2\xi. \tag{6}$$

The density operator for the field of mode 1 conditioned on one-photon measurement for mode 2 is

$$\hat{\rho}_1=\mathcal{N}\,{}_2\langle 1|\hat{\rho}_{\mathrm{out}}|1\rangle_2. \tag{7}$$

Throughout the paper, $\mathcal{N}$ denotes the appropriate normalization factor. For the case of Eq. (7),

$$\mathcal{N}=\frac{1}{{}_2\langle 1|\mathrm{Tr}_1[\hat{\rho}_{\mathrm{out}}]|1\rangle_2}=\frac{[(m_1+1)(m_2+1)]^{3/2}}{2(m_1m_2-1)}. \tag{8}$$

With the knowledge of the one-photon Fock-state expectation value of the displacement operator [20,22]

$$\langle 1|\hat{D}(-\xi)|1\rangle=e^{-|\xi|^2/2}(1-|\xi|^2),$$

the density operator is found to be

$$\hat{\rho}_1=\frac{\mathcal{N}}{\pi^2}\int C(\eta,\xi)\hat{D}_1(-\eta)e^{-|\xi|^2/2}(1-|\xi|^2)d^2\eta\, d^2\xi.$$

The characteristic function is then easily obtained using the identity $\mathrm{Tr}[\hat{D}(\zeta)\hat{D}(-\eta)]=\pi\delta^{(2)}(\zeta-\eta)$:

$$\begin{aligned}C_1(\zeta)=&\left[1-\frac{c_1^2(m_2+1)\zeta_r^2}{(m_1+1)(m_1m_2-1)}\right.\\&\left.-\frac{c_2^2(m_1+1)\zeta_i^2}{(m_2+1)(m_1m_2-1)}\right]\exp\left[-\frac{1}{2}\left(n_1-\frac{c_1^2}{m_1+1}\right)\zeta_r^2\right.\\&\left.-\frac{1}{2}\left(n_2-\frac{c_2^2}{m_2+1}\right)\zeta_i^2\right].\end{aligned} \tag{9}$$

By Fourier transformation of the Weyl characteristic function [23], we obtain the Wigner function. Now, the first point we are interested in is the negativity of the Wigner function. It is clear that the Fourier transform of Eq. (9) has the largest negativity (if any exists) at the origin of phase space and the value of the Wigner function at the point is

$$W_1(0)\propto\frac{B-1}{(T+1)B+R}+\frac{A-1}{(T+1)A+R}, \tag{10}$$

which has been obtained by substituting the parameters (5). It is obvious that if $A>1$ or $B>1$, i.e., the incoming Gaussian field is not squeezed, $W_1(0)$ is positive everywhere. In order to find the exact condition for negativity in the Wigner function, we assume that $A<1$, $B>1$, and introduce positive parameters $x=(A+1)/(1-A)$ and $y=(B+1)/(B-1)$. Then the right-hand side (RHS) of Eq. (10) becomes

$$\frac{2T-x+y}{(T-x)(T+y)}$$

whose denominator is always negative. The numerator becomes positive when the transmittivity satisfies

$$T>\frac{AB-1}{(1-A)(B-1)}, \tag{11}$$

which always holds when the incoming Gaussian field is pure $AB=1$ (in other words, if the incoming Gaussian field is a pure squeezed state, the Wigner function always shows negativity by subtracting a photon from it).

The P function of the field may be obtained using the relation between its characteristic function $C_1^{(p)}$ and the Weyl characteristic function [23]:

$$C^{(p)}(\zeta)=C(\zeta)e^{|\zeta|^2/2}. \tag{12}$$

With use of the characteristic function (9) and general Gaussian integration, we find that the P characteristic function is integrable when $(n_i-1)(m_i+1)-c_i^2>0$ for $i=1,2$. By substituting the parameters (5), we find the condition equivalent to $2T(A-1)>0$ and $2T(B-1)>0$. So if the incoming field is squeezed, it is not possible to integrate $C^{(p)}$ and no P function exists. Considering the positivity of the P function, after a little algebra with Fourier transformation of the P characteristic function, we find that the P function is positive as long as it exists in this case. We conclude that the single-photon subtracted field is nonclassical (in the sense of a lack of an acceptable P function) provided the original incoming field is squeezed. (However, the Wigner function does not necessarily show negativity for all those nonclassical states unless the incoming Gaussian field was pure.) Unless the incoming Gaussian field is nonclassical we cannot generate a nonclassical state by subtracting a photon from it.

This seemingly trivial result is not obvious at all as contrasted by the nonclassicality of a field by adding a photon into a Gaussian field [25,26]. In distinction to the case of subtracting a photon, the photon-added Gaussian state always shows negativity at the origin of the phase space [26–28]. By adding a photon, a highly classical state such as a high-temperature thermal state becomes nonclassical, showing negativity in its Wigner function. The realization of such a photon-added state is beyond the scope of the current work but we may think of a possibility within cavity quantum electrodynamics or the phonon state of a driven ion in a cavity [28].

We now introduce the coherent-superposition state [24]

$$|\psi\rangle = \mathcal{N}(|\alpha\rangle - |-\alpha\rangle), \tag{13}$$

where $\mathcal{N}=1/\sqrt{1-\exp[-2\alpha^2]}$, to assess its fidelity to the photon-subtracted Gaussian state. It is straightforward to calculate the characteristic function of the coherent-state superposition from Eq. (13) [6]. The closeness of two states, one of which is a pure state $|\phi\rangle$ and the other (pure or mixed) is represented by its density operator $\hat{\rho}_1$ is measured by the fidelity $\mathcal{F}$:

$$\mathcal{F} = \langle\phi|\hat{\rho}|\phi\rangle = \frac{1}{\pi}\int d^2\zeta\, C_\phi(\zeta)C_\rho(\zeta) \tag{14}$$

where the subscripts refer to the respective states.

The fidelity between $\hat{\rho}_1$ and the coherent-state superposition (13) has been calculated from Eqs. (1), (13), and (14) and plotted in Fig. 1. The incoming Gaussian field has been assumed a pure squeezed field. In Fig. 1, the solid line is the optimized fidelity between the photon-subtracted state and the ideal coherent-state superposition by an ideal single-photon detector. The fidelity is very high as $\mathcal{F}>0.99$ regardless of the transmittivity of the beam splitter when an ideal single-photon detector is used. The optimized amplitude of the ideal coherent-state superposition is $\alpha=1.16$ for the transmittivity close to unity. If the transmittivity T gets smaller, the amplitude of the ideal coherent-state superposition, which maximizes the fidelity, also becomes smaller. For example, the amplitude will be $\alpha=1.02$ (1.09) for $T=0.8$ (0.9). However, the fidelity is not sensitive to the transmittivity of the beam splitter as shown in Fig. 1 because the single-photon detector successfully subtracts only one photon from the Gaussian state regardless of the transmittivity of the beam splitter. In fact, the fidelity gets slightly better as the transmittivity becomes smaller, due to the fact that both of the states are reduced to the exact single-photon state as $T\to 0$.

It is interesting that the fidelity between the photon-subtracted field $\hat{\rho}_1$ and the coherent-state superposition is very high. This could have been guessed from their photon-number distributions. The squeezed vacuum is a state with only an even number of photons [6] while the coherent-state superposition (13) is a state with only an odd number of photons [12]. By subtracting one photon from the squeezed state, the two states may become closer to each other. We see that the photon-subtracted squeezed field is close to the coherent-state superposition of small amplitudes. One reason can be found again in their photon-number distributions. The photon-number distribution of $|\psi\rangle$ peaks around $|\alpha|^2$ while that of $\hat{\rho}_1$ is a monotonically decreasing function with regard to the photon number. Thus, when α is small, the distributions become similar to each other. Of course, this check of the photon-number distributions gives only a hint as the photon-number distribution does not necessarily convey all the coherence properties of a quantum field.

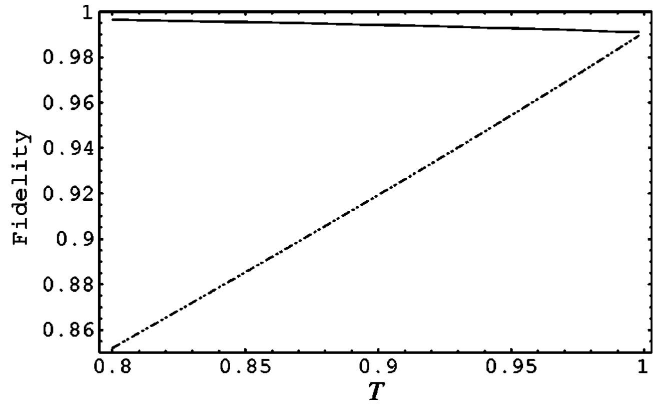

FIG. 1. The fidelity between the photon-subtracted state and the ideal coherent-state superposition with an ideal single-photon detector (solid line) and a threshold detector (dotted line). The initial squeezing parameter is $\exp(2s)=2.36$ and the x axis is the transmittivity of the beam splitter $T=t^2$. The amplitude α of the ideal coherent-state superposition is optimized for the maximum fidelity. The optimized amplitude α ranges between 1.02 (when $T=0.8$) and 1.16 (when $T\to 1$).

III. EXPERIMENTAL REALITY

As can be seen in Fig. 1, the fidelity between the ideal coherent-state superposition and the photon-subtracted state is not so sensitive to reflectivity of the beam splitter. This seemingly good result is due to an ideal single-photon detector assumed for the photon-subtracted state $\hat{\rho}_1$. As mentioned, the state (7) is what is wanted to achieve but the available high-efficiency photodetector is not able to discern one and any number of photons. Thus the state experimentally generated using such a threshold photodetector is

$$\hat{\rho}_a = \mathcal{N}\sum_{n=1}^{\infty} {}_2\langle n|\hat{\rho}_{\rm out}|n\rangle_2. \tag{15}$$

Consider the density operator for mode 1 of the output field,

$$\hat{\rho}_t = {\rm Tr}_2[\hat{\rho}_{\rm out}] = \sum_{n=0}^{\infty} {}_2\langle n|\hat{\rho}_{\rm out}|n\rangle_2. \tag{16}$$

It is then clear from Eqs. (15) and (16) that

$$\hat{\rho}_a = \mathcal{N}(\hat{\rho}_t - {}_2\langle 0|\hat{\rho}_{\text{out}}|0\rangle_2) \tag{17}$$

where

$$\hat{\rho}_t = \frac{1}{\pi}\int C_{\text{out}}(\eta,0)\hat{D}_1(-\eta)d^2\eta \tag{18}$$

and

$${}_2\langle 0|\hat{\rho}_{\text{out}}|0\rangle_2 = \frac{1}{\pi^2}\int C_{\text{out}}(\eta,\xi)e^{-|\xi|^2/2}\hat{D}_1(-\eta)d^2\eta\, d^2\xi. \tag{19}$$

Using $C_{\text{out}}(\eta,\xi)$ as we have already discussed, we find the characteristic function $C_a(\zeta)$ for $\hat{\rho}_a$:

$$C_a(\zeta) = \mathcal{N}e^{-(n_1\zeta_r^2+n_2\zeta_i^2)/2}\left[1 - \frac{2}{\sqrt{(m_1+1)(m_2+1)}} \times \exp\left(\frac{c_1^2}{2(m_1+1)}\zeta_r^2 + \frac{c_2^2}{2(m_2+1)}\zeta_i^2\right)\right]. \tag{20}$$

The normalization factor is calculated as

$$\mathcal{N} = \frac{\sqrt{(m_1+1)(m_2+1)}}{\sqrt{(m_1+1)(m_2+1)} - 2}.$$

The Wigner function obtained by Fourier transformation of the characteristic function (20) is what Wenger *et al.* would have reconstructed [13] if the detection efficiency of their experiment had been perfect and the modal purity unity.

Let us next consider the negativity of the Wigner function. By inspection of the characteristic function, we realize that the Wigner function has the largest negativity (if any) at the origin of the phase space and the value of the Wigner function at this point is

$$W_a(0) = \frac{2\mathcal{N}}{\pi}\left\{\frac{1}{\sqrt{n_1 n_2}} - \frac{2}{\sqrt{[n_1(m_1+1)-c_1^2][n_2(m_2+1)-c_2^2]}}\right\}. \tag{21}$$

By partly substituting the parameters (5), we find that the Wigner function becomes negative when

$$\frac{2}{\sqrt{(n_1+A)(n_2+B)}} > \frac{1}{\sqrt{n_1 n_2}} \tag{22}$$

which becomes a criterion for the transmittivity

$$T > \frac{4-(A+1)(B+1)}{3(A-1)(B-1)}. \tag{23}$$

For a pure squeezed Gaussian incoming field, the condition becomes $T > 1/3$. It is interesting to note that regardless of the degree of squeezing (provided it is not zero), we can see the negativity in the Wigner function provided the transmittivity is larger than 1/3.

Let us assess the degree of nonclassicality by the P function criterion. With use of the relation (12) between the characteristic functions, we note that the P characteristic function for $\hat{\rho}_a$ is integrable when $T(A-1)>0$ and $T(B-1)>0$. We have checked that the P function is semipositive when it exists and conclude that, for nonzero transmittivity of the beam splitter, if and only if the incoming Gaussian field is squeezed, the any-number photon-subtracted state $\hat{\rho}_a$ is nonclassical. Again, the P function criterion is weaker than the negativity criterion for the Wigner function.

We now consider how close the field obtained using the threshold detector is to the coherent-state superposition (13). The optimized fidelity has been calculated using Eqs. (13), (14), and (20), and plotted in Fig. 1. It tells us that the state which is obtained by subtracting any number of photons is similar to the coherent-state superposition only when the transmittivity of the beam splitter is very high. For example, the fidelity is higher than 90% when $T>0.87$. In this case, the chance of one-photon subtraction is more likely. Note that the optimized amplitude α ranges between 1.02 (when $T=0.8$) and 1.16 (when $T\to 1$) in Fig. 1.

A. Inefficient detection and modal purity

Homodyne detection may be used to reconstruct the Wigner function for the field $\hat{\rho}_a$. Even though homodyne detectors are known for their high efficiency, the overall detection efficiency was about 75% in Wenger *et al.*'s experiment [13]. An imperfect detector is equivalent to a perfect detector with a beam splitter in front [29], where the transmittivity of the beam splitter is determined by the detection efficiency η. From Ref. [30], we note that the characteristic function for the signal field passing through a beam splitter where the other input port is served by the vacuum is

$$C_{\text{im}}(\zeta) = C_a(\sqrt{\eta}\zeta)C_v(\sqrt{1-\eta}\zeta). \tag{24}$$

Substituting Eq. (20) into Eq. (24), we find the characteristic function for the detected field. The Fourier transform of the characteristic function shows its largest negativity at the origin of the phase space and the value there is

$$W_{\text{im}}(0) \propto \frac{1}{\sqrt{vw}} - \frac{1}{\sqrt{(v-R(A-1)/2)(w-R(B-1)/2)}} \tag{25}$$

where $v=T(A-1)\eta+1$ and $w=T(B-1)\eta+1$. Under the assumption $(A-1)(B-1)<0$, this becomes negative when the detection efficiency satisfies

$$\eta > -\frac{1}{2T(A-1)} - \frac{1}{2T(B-1)} - \frac{R}{4T}.$$

In particular, for a pure Gaussian incoming field, the condition becomes

$$\eta > \frac{1+T}{4T}. \tag{26}$$

The RHS is smaller than unity (the detection efficiency $\eta \leqslant 1$) only when $T \geqslant 1/3$. This is in good agreement with the perfect detection case. So, in order to see negativity in the Wigner function, the beam splitter has to have a transmittivity larger than 1/3 first and then the detection efficiency has to satisfy the condition (26). Wenger *et al.* employed a beam splitter with $T\approx 0.88$ in which case the detection has to be larger than a mere 53.4% to see negativity in the Wigner function.

Another important factor which degrades the quantum effect of the photon-subtracted Gaussian state in a real experiment is the modal purity factor [13]. If the dark count rate of the photodetector employed to subtract a photon is non-negligible, the resulting state can be estimated in a mixture of photon-subtracted squeezed state and squeezed state as

$$\xi W(\alpha) + (1 - \xi) W_{\mathrm{sq}}(\alpha) \tag{27}$$

where $W(\alpha)$ is the Wigner function of the photon-subtracted squeezed state, $W_{\mathrm{sq}}(\alpha)$ is the Wigner function of the squeezed state, and ξ corresponds to the modal purity factor, which was 0.7, in Wenger *et al.*'s experiment [13]. The Wigner functions of the photon-subtracted Gaussian state have been plotted for a number of different cases in Fig. 2. It shows that the negativity of the Wigner function disappears when both of the homodyne efficiency η and the modal purity ξ are considered taking relevant experimental values. We suggest that either the homodyne efficiency should be improved from 0.75 to 0.9 or the modal purity factor should be improved from 0.7 to 0.9 to clearly observe the negativity of the Wigner function. In these cases, the minimum negativity will be −0.044 and −0.073, respectively.

IV. REMARKS

In this paper, we are interested in the nonclassicality of a state produced by subtracting photons from a Gaussian field. Subtracting a photon does not transform a classical state into a nonclassical state whereas a nonclassical input remains nonclassical. This is in contrast to the case of adding a photon to a Gaussian field, in which case even a very chaotic field transforms into a nonclassical state [26–28]. The non-Gaussian state obtained by subtracting a photon from a Gaussian field may show large negativity in its Wigner function. The condition to obtain the negativity is analyzed for a realistic case including the mixed-state input, threshold detection, inefficient homodyne detection, and modal purity. The non-Gaussian state analyzed in this paper is compared with a coherent-state superposition which may be extremely useful for fundamental and application reasons. The comparison shows fidelity higher than 90% for the experimentally relevant situation. We compare our analysis with a recent experimental demonstration [13] of the photon-subtracted Gaussian field and suggest that either the homodyne efficiency or the modal purity factor should be improved to around ~0.9 to clearly observe the negativity of the Wigner function.

Note added. Recently, we were made aware of Ref. [31] which considers nonlocality of a photon-subtracted squeezed state.

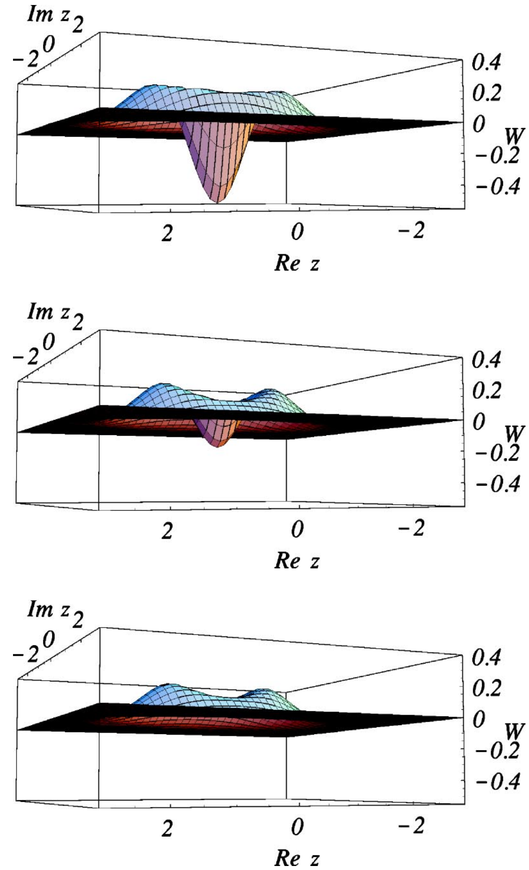

FIG. 2. (Color online) (a) The Wigner function $W(z)$ of a photon-subtracted Gaussian state with a threshold detector for photon subtraction and ideal homodyne detectors for reconstruction of the Wigner function when $T=0.88$ and $\exp(2s)=2.36$. The minimum negativity is found as $W(0,0)=-0.52$. (b) The Wigner function of a photon-subtracted Gaussian state under the same condition as (a) but with homodyne efficiency $\eta=0.75$. The minimum negativity has been reduced to −0.15. (c) The Wigner function of a photon-subtracted Gaussian state under the same condition as (a) but with homodyne efficiency $\eta=0.75$ and with the modal purity factor 0.7. The negativity of the Wigner function has disappeared as $W(0)=0.075$.

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